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**ACTIVE PRECESSION CONTROL SYSTEM OF A SOUNDING ROCKET  
WITH CONTROL TORQUE IN ONLY ONE AXIS**

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**ACTIVE PRECESSION CONTROL SYSTEM OF A SOUNDING ROCKET WITH CONTROL TORQUE IN  
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**ABSTRACT**

A study of an active precession control system aiming to stop the precession motion of a sounding rocket, spin stabilized, is presented. Also, a strategy of displacement of the sensor and the actuator axes is presented, which implies on mechanical lead phase compensation. The spin stabilization permits to use only one actuation plane, which reduces the weight and the cost of the control system.

The control of the precession motion, which is expressed by the Euler angles on the inertial reference frame, is achieved indirectly by the canceling of angular velocities on the body frame, Fig.(1). However, it does not mean to lead the misaligned angle (caused by precession) to zero, because it will be necessary an inertial attitude control system on three axes.

Two types of control law are presented, one linear and another nonlinear, and two types of actuator are considered, the proportional one and the on-off one.

**I INTRODUCTION**

Several sounding rockets or last stage of a launch vehicle are spin stabilized. This procedure, which is a passive attitude control, offers a gyroscopic stiffness to the vehicle and prevents against small disturbances.

Despite of the spin stabilization, if a considerable disturbance acts on the vehicle, it causes two types of motion named precession and nutation. The precession motion can be catastrophic to the mission because it can result in a collision of the last stage of the vehicle with the payload, error of the

final orbit of the payload, and it can result in other problems that can not be predicted. So, a control system that prevent the vehicle against these problems, along with the spin stabilization, is necessary.

An active precession control system is presented in this work. Two types of control law are resulted: linear with on-off actuator and nonlinear (LQR) with proportional actuator, which show their performances in stopping the precession motion with minimum energy, or control torque. The only criteria performance for the two control systems was the settling time established in 5s.

Two strategies, which reduce the number of sensor and actuator of the embedded control system are presented. The first one is the actuation on only one axis, the axis 1. This is possible because the vehicle is spin stabilized. This type of control is employed on several sounding rockets or launchers, and detailed discussion is presented (Ball and Osborne, 1967). The second one is to displace the sensor axis from the actuator axis of an angle  $\alpha$ , as Fig. (2) shows.

The simulations of the control system are carried out on a software Matlab/Simulink.

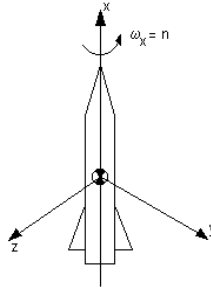


Fig. (1). Rockets Principal axes aligned with the body reference frame.

## II THE EQUATIONS OF MOTION

The rotating motion of a spinning body is described by the Euler Equations(Wie, 1998).

$$J_1 \dot{x}_1 = (J_2 - J_3) \dot{x}_2 \dot{x}_3 + \tau_1 \quad (1a)$$

$$J_2 \dot{x}_2 = (J_3 - J_2) \dot{x}_1 \dot{x}_3 + \tau_2 \quad (1b)$$

$$J_3 \dot{x}_3 = (J_1 - J_2) \dot{x}_1 \dot{x}_2 + \tau_3 \quad (1c)$$

For a body with the form of a sounding rocket, Fig.(1), which has a form of a pencil, and considering that the rocket principal axes are aligned with the body reference frame, the Euler's Equations of motion become:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \lambda \\ -\lambda & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u \quad (2)$$

where  $x_1$  and  $x_2$  are the transversal angular velocities of the body reference frame,  $\lambda$  is the relative spin rate  $\lambda = \frac{J - J_3}{J} n$ ; and  $n$  is the spin,  $J$  is the transversal inertia moment and  $J_3$  the longitudinal moment;  $u$  is the control and  $b$  is a constant.

As the aim of the control system is to stop the precession motion, it is necessary to express the angular velocities of the body reference frame in an inertial reference frame. For that, the Euler angles on the sequence 1-2-3 are used (Wie, 1998). So, the kinetic equations on the inertial reference frame, related to angular velocities of the body reference frame, are:

$$\dot{\psi} = (x_1 \sin \phi - \omega_2 \cos \phi) / \cos \theta \quad (3a)$$

$$\dot{\theta} = x_1 \cos \phi + x_2 \sin \phi \quad (3b)$$

$$\dot{\phi} = x_3 + (-x_1 \sin \phi + x_2 \cos \phi) \tan \theta \quad (3c)$$

Fig. (3) shows the relation between the two references frames.

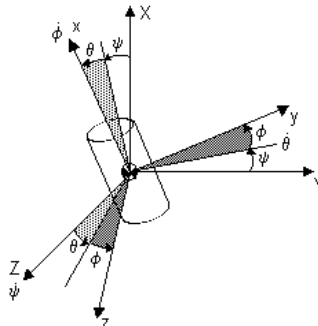


Fig. (3). Body reference frame and Inertial Reference frame related by Euler angles

The following figure shows the precession motion of the sounding rocket

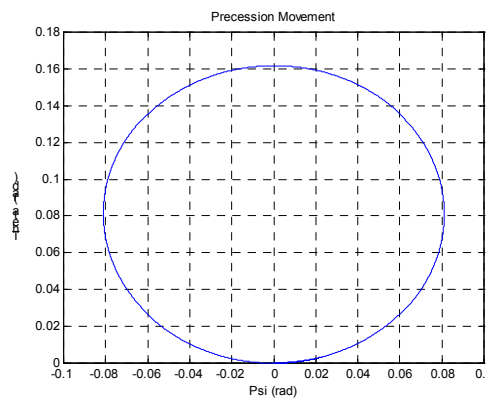


Fig. (4). Uncontrolled Precession Motion with 4.65° of amplitude

### III SENSOR AND ACTUATOR MISALIGNMENT

Misalignment of the sensor and actuator implies in a mechanical lead phase compensation, which is represented by Fig.(2). It implies in measuring the projection of the transversal angular

velocity on the axes 1 e 2, simultaneously, and it results on the following equations for the gains, Eqs. (4) (Guilherme, 2001):

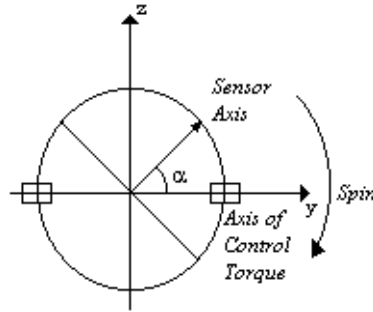


Fig. (2). Cross section centered on the mass center of the rocket displacement of the sensors and actuators.

$$k_1 = k \cos \alpha \quad (4a)$$

$$k_2 = k \sin \alpha \quad (4b)$$

The control law which will be used all over the control systems presented in this text is as follows

$$u(t) = -k\omega_{12} \quad (5)$$

or

$$u(t) = -k_1\omega_1 - k_2\omega_2 \quad (6)$$

where

$$\omega_1 = (\cos \alpha)\omega_{12} \quad (7a)$$

$$\omega_2 = (\sin \alpha)\omega_{12} \quad (7b)$$

Introducing the Eqs.(4) and Eqs.(7) in the Eq.(6) leads to the Eq.(5). This proves that the dislocated sensor measures the two projections of the transversal angular velocities.

From Eqs.(4) can be seen that the gains became dependent on the angle between the sensor axis and the actuator axis. So, since the gains were found from LQR, the angle can be determined by the Eqs.(4).

#### IV PROPORTIONAL CONTROL SYSTEM

Introducing the control law Eq.(6) in Eq.(2) lead to

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & \lambda - k_2 \\ -\lambda & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (8)$$

where the control was incorporated by its relations on the state variables.

Because the differential equations are coupled, the control torque will act on the two axes simultaneously.

The Linear Quadratic Regulator (LQR) technique was used to find the optimal gains,  $k_1$  and  $k_2$ , of the proportional control system, from the following Performance Index (PI):

$$J = \int_0^{\infty} (x^T Q x + b u^2) dt$$

where  $\mathbf{x}$  are the states to be controlled,  $Q = [q_y^2 \ 0, 0 \ q_z]$  is a weight matrix for the states,  $\mathbf{u}$  is the control torque and  $b$  its constant weight.

Fig.(5) presents a block diagram which illustrates the optimal control law.

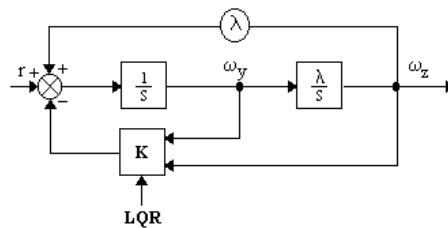


Fig. (5). Block Diagram with optimal control law (LQR)

For a settling time of 5s, resolving the Algebraic Ricatti Equation (ARE) for Eqs.(2), for the data of Table 1, the gains are found to be:  $k_1 = 1.6$  and  $k_z = -0.08$

From Eqs.(4), these gains result in  $3^\circ$ . Fig(7) shows the controlled angular velocities of the body reference frame, the controlled precession motion, and the resulted control torque, of the proportional control system.

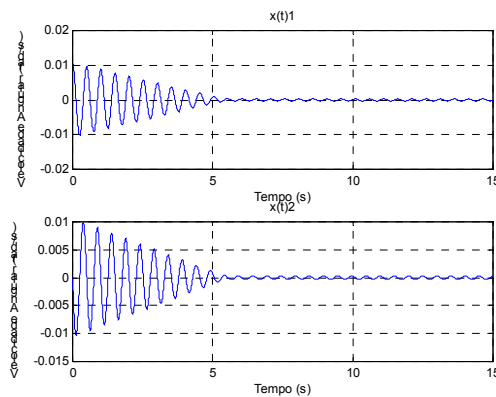


Fig.(7a). Controlled Angular Velocities of the Body Reference frame with nonlinear control.

Settling time of 5s.

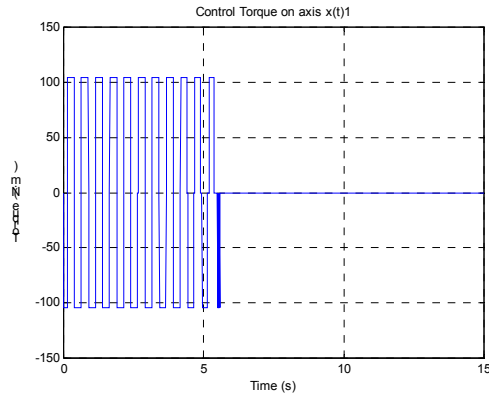


Fig.(7b). Control Torque Amplitude of 103.93 N.m

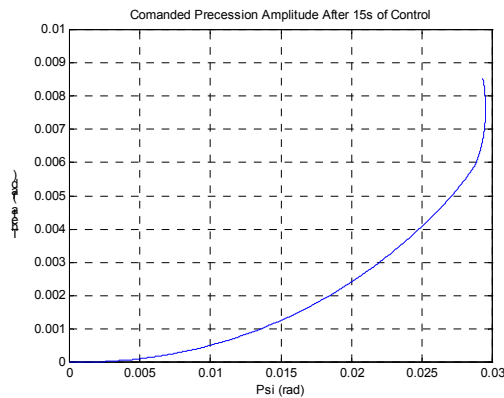


Fig.(7c). Controlled precession motion stoped with 1.78° of amplitude.

It must be emphasized that the angle (3°) found by the gains is extremely related to the spin of the vehicle.

## V ON-OFF CONTROL SYSTEM

This control system consists on the use of the relay with dead zone. Fig.(6) shows the block diagram of the control system.

Despite of the optimal control system above mentioned, it presents the problem of the actuator being proportional. Proportional thrusters, whose valves open proportionally to the commanded thrust level, are not much used in practice. Mechanical considerations prevent the proportional valve operation largely. On-off control law has been developed to permit the use of on-off valves.

In this control system the control law is:

$$u(t) = M \text{sign}(-k_1 \omega_1 - k_2 \omega_2) \quad (9)$$

As can be see from the Eq.(9), now the amplitude of the control torque is restricted.

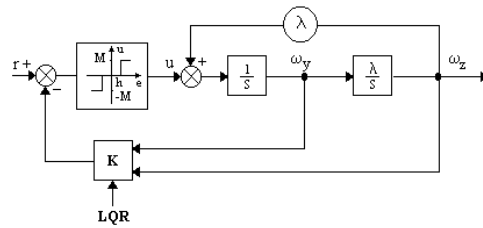


Fig. (6). Control system using the LQR optimal gains and relay with dead zone.

As the criteria performance is only the settling time, the objective of the on-off control system was to achieve the amplitude of torque that stops the precession motion in the same range of time. Fig. (8) shows the controlled angular velocities of the body reference frame, the controlled precession motion, and the resulted control torque, of the on-off control system.

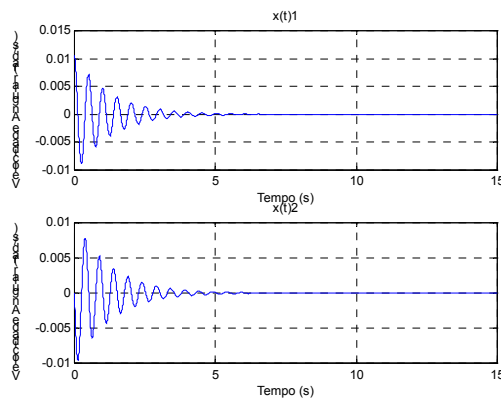


Fig.(8a). Controlled Angular Velocities of the Body Reference frame with linear control. Settling time of 5s.

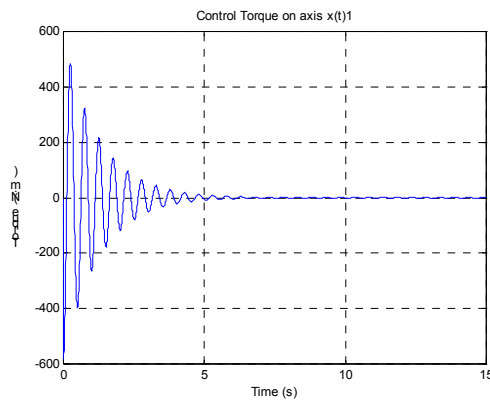


Fig.(8b). Control Torque 1th Peak of 582 N.m.



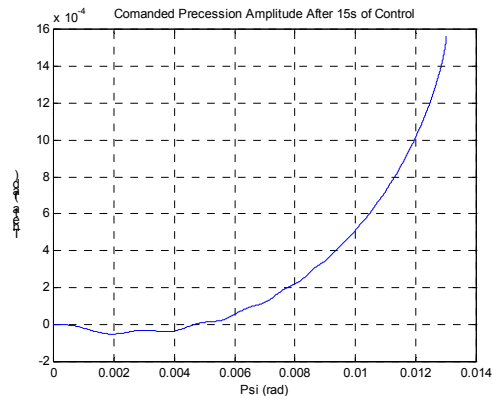


Fig.(8c). Controlled precession motion stoped with  $0.75^\circ$  of amplitude.

## VI CONCLUSION

Although the on-off control system presents an amplitude of the precession motion bigger than the proportional one, it shows the advantage of an amplitude of the control torque minor than the proportional one, for the same energy, because the time of the control actuation is the same. Besides, depending on the mission, the two amplitudes of precession motion can be within the accepted limits.

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## APPENDIX I

Table 1

	1	2	3
Inertia Moments (kg.m <sup>2</sup> )	34641	34641	360.93
Initial conditions of angular velocities of the body reference frame (rad/s)	0.0106	0	4pi
Initial conditions of Euler angles of the inertial reference frame (rad)	Psi=0	Theta=0	Phi=0

Source: Guilherme, M. S. (2001)

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